Kambala Church of England Girls' School

Trial Higher School Certificate Examination , 2000

Year 12

August, 2000

MATHEMATICS

3/4 UNIT

Time Allowed : 2 hours (plus 5 minutes reading time)

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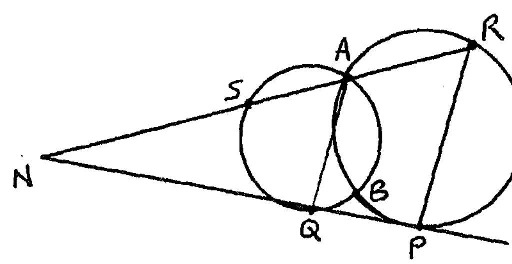
Instructions :

- ALL questions may be attempted.
- ALL questions are of equal value.
- All necessary working should be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Approved scientific calculators and drawing templates may be used.
- A Table of Standard Integrals is contained at the end of the examination paper.
- Start each question in a NEW BOOK.
- This is a Trial Paper only, and does NOT necessarily reflect either the content or format of the final HSC Examination.

Question 1 :

(a) Solve $\dfrac{x^2-1}{x} > 0$.

(b) Two circles intersect at A and B and a common tangent touches them at P and Q.
PR // QA.
RA is produced to cut the other circle at S and the tangent at N.
Prove that PRSQ is a cyclic quadrilateral.



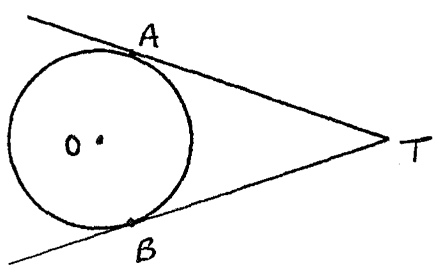
(c) Find $\displaystyle\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$.

(d) Find the equation of the two lines through the point (5,3) which make acute angles of $\dfrac{\pi}{4}^c$ with the line $2x-y+2=0$.

Question 2 : (Start a NEW BOOK)

(a) TA and TB are tangents to the circle drawn below, with centre O.

Prove that : $\dfrac{\angle OAB}{\angle ATB} = \dfrac{1}{2}$.



(b) Find the general solution of the equation $\sin 2\theta = \sin\theta$, θ measured in radians.

(c) Prove by the Principle of Mathematical Induction that $3^{3n} + 2^{n+2}$ is a multiple of 5 for all positive integers n.

Question 3 : (Start a NEW BOOK)

(a) The point P(1,6) divides the interval AB in the ratio m:n.
If A = (7,0) and B = (3,4), find the value of the ratio m:n.

(b) Find $\frac{d}{dx}(x\sin^{-1}2x+\frac{1}{2}\sqrt{1-4x^2})$.

Hence evaluate $\int_0^{\frac{1}{2}} \sin^{-1}2x\,dx$.

(c) (i) If $t=\tan\frac{\theta}{2}$, find $\cos\theta$ and $\sin\theta$ in terms of t.
(ii) Hence solve the equation $3\sin\theta+4\cos\theta=5$ for values of θ in the range $0°\le\theta\le 360°$.

Question 4 : (Start a NEW BOOK)

(a) Evaluate the following definite integrals using the substitutions given :

(i) $\int_0^3 \frac{x}{\sqrt{4-x}}\,dx$ substitute $u=4-x$.

(ii) $\int_0^2 \frac{dx}{(4+x^2)^2}$ substitute $x=2\tan\theta$.

(b) The polynomial $P(x)=ax^3+bx^2-8x+3$ has a factor of $(x-1)$ and leaves a remainder of 15 when divided by $(x+2)$.

(i) Find the values of a and b .
(ii) Hence, factorise $P(x)$ fully and sketch the curve.
(iii) Determine the set of values of x for which $P(x)>0$.

Question 5 : (Start a NEW BOOK)

(a) $P(2ap,ap^2)$ is any point on the parabola $x^2 = 4ay$. S is the focus (0,a) . The tangent to the parabola at P meets the Y-axis in M. The perpendicular to the tangent PM from S meets PM in N. Find :

(i) the co-ordinates of M and N.
(ii) the co-ordinates of the midpoint K of MN.
(iii) the equation of the locus of K as P varies.

(b) A circular oil slick lies on the surface of a body of calm water. If its area is increasing at the rate of 1500 m^2/h , at what rate is its circumference increasing when the radius of the slick is 1250 m.

Question 6 : (Start a NEW BOOK)

A stone is thrown horizontally with a velocity of 20 m/s from the top of a tower 100 m high. Assuming no air resistance, and that the acceleration due to gravity, $g \approx 10\ m/s^2$;

(i) express x and y in terms of t.
(ii) find the equation of the trajectory.
(iii) find how long the stone takes to reach the ground .
(iv) find how far from the foot of the tower the stone strike the ground .
(v) find the velocity and direction of the stone on impact with the ground .

Question 7 : (Start a NEW Page)

(a) Define Simple Harmonic Motion.

(b) A particle moves from the origin, O with velocity (2p) m/s, and is subject to a retardation of its motion equal to q times its distance x from the origin (q>0).
(Note : retardation means negative acceleration)

(i) Show that the distance it travels, before coming to rest is $\frac{2p}{\sqrt{q}}$ metres.

(ii) Find the time when the particle first comes to rest.

(iii) Find where the particle is after $\frac{\pi}{4\sqrt{q}}$ seconds.

END OF EXAM

Year 12 3U THSC 2000 (PSJ)

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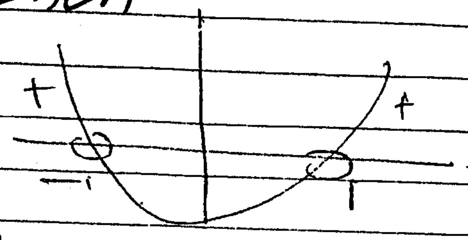
Q1 (a) $\dfrac{x^2-1}{x} > 0$

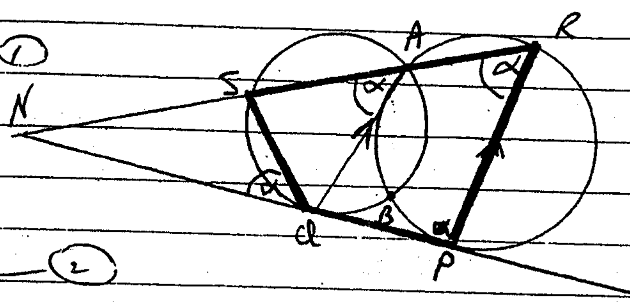


$\times x^2$: $x^2 - 1 > 0$

$(x-1)(x+1) > 0$

2 $\boxed{x > 1,\ x < -1}$ — 2

(b) $\angle SAQ = \angle ARP = \alpha$ — ①
(corresp. ∠s = in ∥ line (PR, AQ))



$\angle SAQ = \angle SQN = \alpha$ — ②

3 (angle between a tangent and a chord = angle in the alternate segment)

$\therefore \angle SQN = \angle SRP$

$\therefore$ PQRS is a cyclic quad (ext ∠ = inter. opp ∠)

(c) $\displaystyle\int_0^{2/3} \frac{dx}{4+9x^2} = \frac{1}{9}\int_0^{2/3} \frac{dx}{\frac{4}{9}+x^2} = \frac{1}{9}\cdot\frac{3}{2}\left[\tan^{-1}\left(\frac{3x}{2}\right)\right]_0^{2/3}$

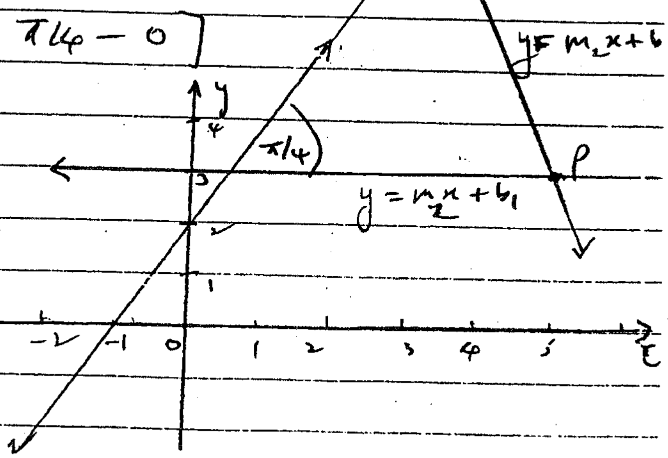
3 $= \frac{1}{6}\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$ $\frac{\pi}{4}$

$= \frac{1}{6}\left[\pi/4 - 0\right]$

$= \boxed{\pi/24}$

(d) $2x - y + 2 = 0$

x	0	-1
y	2	0



$m_1 = 2$

$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow$

Q1(d) $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\tan \pi/4 = \left|\frac{2 - m_2}{1 + \cancel{2} m_2}\right|$

$1 = \left|\frac{2 - m_2}{1 + 2m_2}\right|$

4

$1 + 2m_2 = \pm (2 - m_2)$

$1 + 2m_2 = 2 - m_2$ $\quad\quad$ $1 + 2m_2 = -2 + m_2$

$3m_2 = 1$ $\quad\quad$ $m_2 = -3$

$m_2 = 1/3$

$m_2 = \frac{1}{3}, (5, 3)$

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{1}{3}(x - 5)$

$3y - 9 = x - 5$

$x - 3y + 4 = 0$

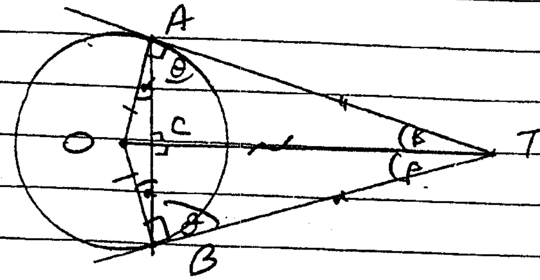
$m_2 = -3, (5, 3)$

$y - 3 = -3(x - 5)$

$y - 3 = -3x + 15$

$3x + y - 18 = 0$

Q2 (a) Let $\angle OAB = \alpha$
and $\angle ATB = 2\beta$.
and $\angle BAT = \theta$



$\Delta OAT \equiv \Delta OBT$ (RHS)
$\therefore \angle ATO = \angle BTO = \beta$
(Corresp. ∠s of ≡ Δs are =)

$\Delta ACT \equiv \Delta BCT$ (SAS)
$\therefore \angle ACT = \angle BCT = 90°$ (= supp. ∠s of a st. angle)

4

In ΔOAT, $\alpha + \theta = 90°$ (compl. ∠s of a L)
In ΔCAT, $\beta + \theta = 90°$ (∠ sum of a Δ)
$\therefore \alpha = \beta$
$\therefore \angle ATB = 2\beta = 2\alpha = 2 \times \angle OAB$.
$\therefore \dfrac{\angle OAB}{\angle ATB} = \dfrac{1}{2}$

(b) $\sin 2\theta = \sin\theta$
$2\sin\theta\cos\theta = \sin\theta$
$\sin\theta(2\cos\theta - 1) = 0$.
$\therefore \sin\theta = 0$ or $\cos\theta = \frac{1}{2}$

3

$\theta = n\pi$ or $\theta = 2n\pi \pm \pi/3$

(c) $3^{3n} + 2^{n+2}$ is a multiple of 5.
S1 $n = 1$ $3^{3n} + 2^{n+2} = 3^3 + 2^3 = 27 + 8 = 35 = 5 \times 7$
$\therefore$ True for $n = 1$

S2 $n = k$: $3^{3k} + 2^{k+2} = 5P$
$\therefore 3^{3k} = 5P - 2^{k+2}$ —— (A)

3

S3: $n = k+1$ $3^{3(k+1)} + 2^{(k+1)+2} = 5Q$ —— (B)

(B). LHS $= 3^{3k+3} + 2^{k+3}$
$= 27(3^{3k}) + 2(2^{k+2})$
$= 27(5P - 2^{k+2}) + 2(2^{k+2})$
$= 5(27P) - 2^{k+2}(27 - 2)$

Q2 (c) $LHS = 5(27p) - 2^{k+1}(25)$

$$= 5\left[27p - 5(2^{k+1})\right]$$

$$= 5Q$$

$$= RHS$$

$\therefore$ By Induction true for all +ve integral n.

Q3 (a) $A(7, 0)$ $B(3, 4)$

$m : n$

$P = \left(\frac{3m + 7n}{m+n}, \frac{4m + 0.n}{m+n} \right)$

$\therefore P = (1, 6)$

$\therefore \frac{3m + 7n}{m+n} = 1$

3

$3m + 7n = m + n$

$2m = -6n$

$m = -3n$

$\therefore \frac{m}{n} = \frac{-3}{1}$

$\therefore m : n = -3 : 1$

(b) $\frac{d}{dx}\left(x \cdot \sin^{-1} 2x + \frac{1}{2}\sqrt{1-4x^2} \right)$ $(1-4x^2)^{1/2}$

$= \sin^{-1} 2x \cdot 1 + x \cdot \frac{2}{\sqrt{1-4x^2}} + \frac{1}{2} \cdot \left(\frac{1}{2}\right) \cdot (1-4x^2)^{-\frac{1}{2}} \cdot -8x$

$= \sin^{-1}(2x) + \cancel{\frac{2x}{\sqrt{1-4x^2}}} - \cancel{\frac{2x}{\sqrt{1-4x^2}}}$

$= \sin^{-1}(2x)$

4

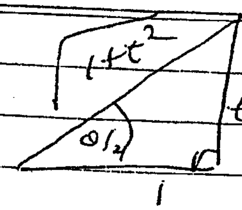
$\therefore \int_0^{1/2} \sin^{-1}(2x)\, dx = \left[x \sin^{-1}(2x) + \frac{1}{2}\sqrt{1-4x^2} \right]_0^{1/2}$

$= \left[\frac{1}{2} \sin^{-1}(1) + 0 \right] - \left[0 + \frac{1}{2} \right]$

$= \frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{1}{2}$

$= \frac{1}{2}\left(\frac{\pi}{2} - 1 \right)$

Q3(c) (i) $t = \tan\frac{\theta}{2}$



$\tan\frac{\theta}{2} = \frac{t}{1}$

$\sin\frac{\theta}{2} = \frac{t}{\sqrt{1+t^2}}$

$\cos\frac{\theta}{2} = \frac{1}{\sqrt{1+t^2}}$

$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$

$= \frac{1}{1+t^2} - \frac{t^2}{1+t^2}$

$\cos\theta = \frac{1-t^2}{1+t^2}$

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$= 2\frac{t}{\sqrt{1+t^2}} \cdot \frac{1}{\sqrt{1+t^2}}$

$\sin\theta = \frac{2t}{1+t^2}$

2

(ii) $3\sin\theta + 4\cos\theta = 5$

$3\left(\frac{2t}{1+t^2}\right) + 4\left(\frac{1-t^2}{1+t^2}\right) = 5$

$6t + 4 - 4t^2 = 5 + 5t^2$

$9t^2 - 6t + 1 = 0.$

$(3t-1)^2 = 0$

$t = 1/3$

$\tan\frac{\theta}{2} = \frac{1}{3}.$

$\frac{\theta}{2} = 18°26'$

$\therefore \theta = 36°52'$

3

Q4 (a) (i) $\int_0^3 \frac{x}{\sqrt{4-x}}\,dx = I$

$u = 4-x$
$du = -dx$

x	0	3
u	4	1

$\therefore I = \int_4^1 \frac{4-u}{\sqrt{u}} - du$

$= +\int_1^4 \frac{4}{\sqrt{u}} - \sqrt{u}\,du$

$= +\int_1^4 4u^{-1/2} - u^{1/2}\,du$

$= \left[\frac{4u^{1/2}}{1/2} - \frac{u^{3/2}}{3/2}\right]_1^4$

$= \left[8\sqrt{u} - \frac{2}{3}\sqrt{u^3}\right]_1^4$

$= \left(8\sqrt{4} - \frac{2}{3}\sqrt{4^3}\right) - \left(8\sqrt{1} - \frac{2}{3}\sqrt{1^3}\right)$

$= \left(16 - \frac{16}{3}\right) - \left(8 - \frac{2}{3}\right)$

$= 8 - \frac{14}{3}$

$= 10/3$

$I = 3\frac{1}{3}$

Qvcd, (ii) $\quad I = \int_0^2 \frac{dx}{(4+x^2)^{\frac{3}{2}}}$

$x = 2\tan\theta$

$dx = 2\sec^2\theta \, d\theta$

x	0	2
θ	0	$\pi/4$

$$I = \int_0^{\pi/4} \frac{2\sec^2\theta \, d\theta}{4 \cancel{8} \sec^3\theta}$$

$$= \frac{1}{4} \int_0^{\pi/4} \cos\theta \, d\theta$$

$$= \frac{1}{4} \Big[\sin\theta \Big]_0^{\pi/4}$$

$$= \frac{1}{4} \left(\sin \pi/4 - \sin 0 \right)$$

$$= \frac{1}{4} \cdot \left(\frac{1}{\sqrt{2}} - 0 \right)$$

3 $\quad = \frac{1}{4\sqrt{2}}$

$$I = \frac{\sqrt{2}}{8}$$

Q4 (b) $f(x) = x^3 - 4x^2 + 3x + 2 = 0$

$\alpha + \beta + \gamma = -b/a = 4$
$\alpha\beta + \beta\gamma + \gamma\alpha = c/a = 3$
$\alpha\beta\gamma = -d/a = -2$

3

(i) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{3}{-2} = -\frac{3}{2}$

(ii) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$
$= (4)^2 - 2(3)$
$= 16 - 6$
$= 10$

Q4 (b) (i) $f(x) = ax^3 + bx^2 - 8x + 3$

$(x-1)$ is a factor of $f(x)$, $\therefore f(1) = 0$

$f(1) = a(1)^3 + b(1)^2 - 8(1) + 3$
$0 = a + b - 8 + 3$
$a + b = 5$ ——— (1)

remainder $= 15$ when $\div$ by $(x+2)$, $\therefore f(-2) = 15$.

$15 = a(-2)^3 + b(-2)^2 - 8(-2) + 3$
$\cancel{15} = -8a + 4b + \cancel{16} + \cancel{3}$ 4
$8a - 4b = 4$
$2a - b = 1$ ——— (2)

3

(1) + (2):
$3a = 6$
$a = 2$

Sub $a = \frac{1}{2}$ into (2):
$2(2) - b = 1$
$4 - b = 1$
$b = 3$

(ii) $f(x) = 2x^3 + 3x^2 - 8x + 3 = (x-1)(2x^2 + 5x - 3)$

$$\begin{array}{r} 2x^2 + 5x - 3 \\ x - 1 \overline{\smash{)}\, 2x^3 + 3x^2 - 8x + 3} \\ 2x^3 - 2x^2 \\ \hline 5x^2 - 8x \\ 5x^2 - 5x \\ \hline -3x + 3 \\ -3x + 3 \\ \hline 0 \end{array}$$

$P = -6$
$S = 5$
$F = 6, -1$

Q4 (b)(ii):- $P(x) = 2x^3 + 3x^2 - 8x + 3$

$= (x-1)(2x^2+5x-3)$

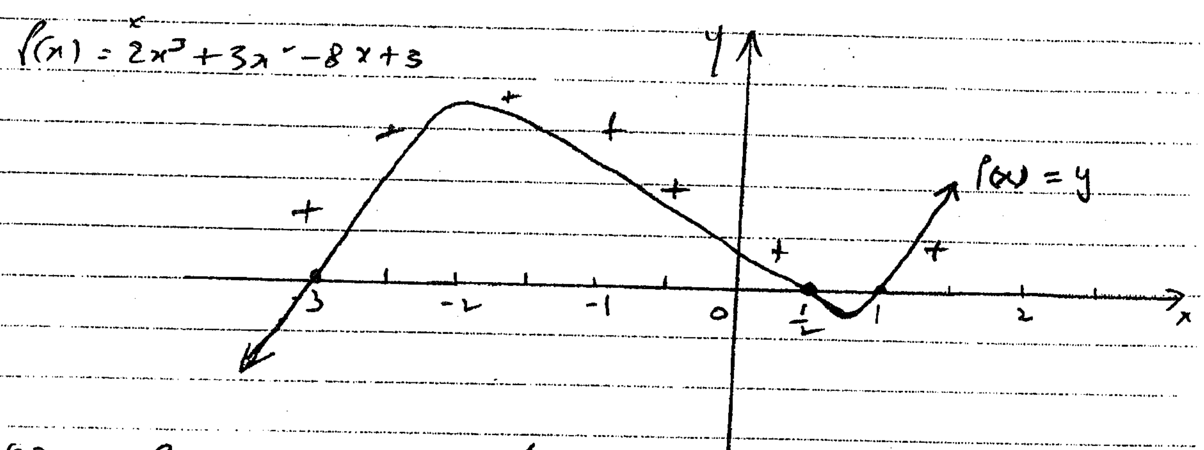
$2x^2+5x-3 = 2x^2 + 6x - 1x - 3$

$= 2x(x+3) - 1(x+3)$

$= (x+3)(2x-1)$

$\therefore P(x) = (x+3)(2x-1)(x-1)$

2



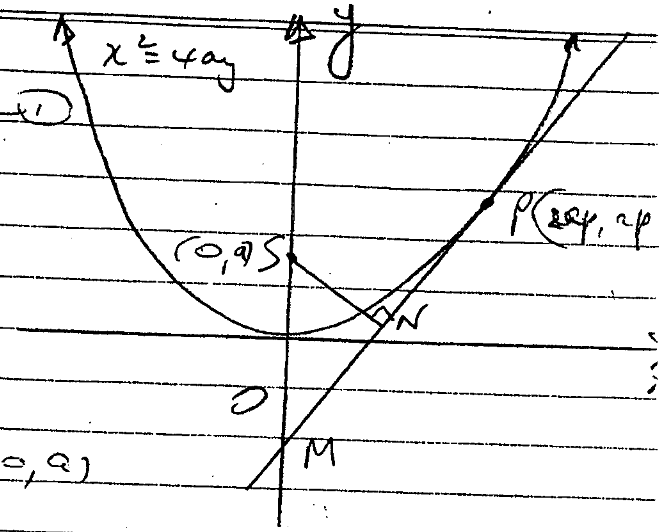
(iii) $P(x) > 0$, from the graph occurs when

$-3 < x < \frac{1}{2}$ & $x > 1$

1

Q5 (a) (i) Tangent at P:

$y - px + ap^2 = 0$ —— (1)



At M, $x = 0$

$y = -ap^2$

2

$\therefore M = (0, -ap^2)$

SN: $m = -\frac{1}{p}$, $S = (0, a)$

$y - a = -\frac{1}{p}(x - 0)$

$py - pa = -x$

$x + py = pa$ —— (2)

To find N, solve (1) & (2) simultaneously

(1) $\Rightarrow$ $y - px = -ap^2$

$y = px - ap^2$ —— (3)

Sub into (2)

$x + p(px - ap^2) = pa$

[redacted]

[redacted]

[redacted]

[redacted]

Q5 (a)(i) $x + p^2x - ap^3 = ap$

$(1+p^2)x = ap + ap^3$

$\cancel{(1+p^2)}x = ap\cancel{(1+p^2)}$

$x = ap.$

Sub $x = ap$ into (3)

4 $\therefore y = p(ap) - ap^2$

$\therefore y = 0.$

$\therefore \boxed{N = (ap, 0)}$

(ii) K = midpt of MN $N = (ap, 0)$ $M = (0, -ap^2)$

1 $\therefore K = \left(\frac{ap+0}{2}, \frac{0 - ap^2}{2}\right)$

$\boxed{K = \left(\frac{ap}{2}, -\frac{ap^2}{2}\right)}$

(iii) $x = \frac{ap}{2}$ — (1) $\Rightarrow$ $p = \frac{2x}{a}$ — (3)

$y = -\frac{ap^2}{2}$ — (2)

Sub (3) into (2):

2 $y = -\frac{a}{2}\left(\frac{2x}{a}\right)^2$

$= -\frac{a}{2}\left(\frac{4x^2}{a^2}\right)$

$y = -\frac{2x^2}{a}$

$\therefore x^2 = -\frac{1}{2}ay$

$$\frac{\cos A - \cos(A+2B)}{2\sin B} = \sin(A+B)$$

$$LHS = \frac{\cos A - \cos A\cos 2B + \sin A \sin 2B}{2\sin B}$$

$$= \frac{\cos A(1-\cos 2B) + \sin A(2\sin B\cos B)}{2\sin B}$$

$$= \frac{\cos A(2\sin^2 B) + \sin A(2\sin B\cos B)}{2\sin B}$$

$$= \frac{2\cancel{\sin B}(\sin B\cos A + \cos B\sin A)}{2\cancel{\sin B}}$$

$$= \sin(B+A)$$

$$= \sin(A+B)$$

$$= RHS$$

5 (b)

$$\frac{dA}{dt} = 1500\,m^2/h$$

$$r = 1250\,m$$

$$C = 2\pi r$$

$$\frac{dC}{dt} = 2\pi \times \frac{dr}{dt}$$

$$= 2\cancel{\pi} \times \frac{3}{5\cancel{\pi}}$$

$$\frac{dC}{dt} = \frac{6}{5}\,m/h$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

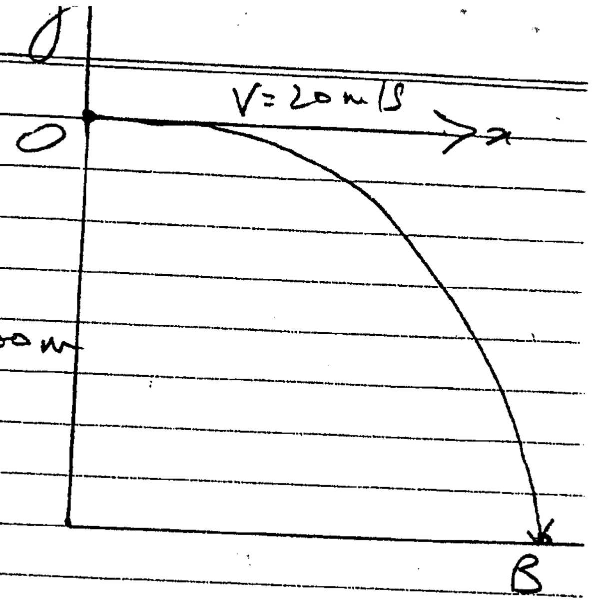
$$1500 = 2\pi \times 1250 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{15\cancel{00}}{25\cancel{00}\pi}$$

$$\frac{dr}{dt} = \frac{3}{5\pi}\,m/h$$

3

Q1 $g = 10\,m/s^2$



(i) $\ddot{x} = 0$

$\dot{x} = V\cos\theta$

$\dot{x} = 20 \times \cos 0$

$\dot{x} = 20$

$x = 20t$

$\ddot{y} = -10$

$\dot{y} = -10t + V\sin\theta$

$\dot{y} = -10t + 20 \times 0$

$\dot{y} = -10t$

$y = -5t^2$

2

(ii) From (i) $x = 20t \Rightarrow t = \frac{x}{20}$

& (ii) $y = -5t^2$

$\therefore y = -5\left(\frac{x}{20}\right)^2$

$= \frac{-5 \cdot x^2}{400}$

$y = \frac{-x^2}{80}$

$\therefore x^2 = -80y$

2

(iii) $y = -100$

$-5t^2 = -100$

$t^2 = 20$

$t = \sqrt{20}$ sec

2

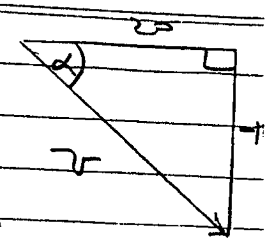
(iv) $t = \sqrt{20}$ $x = 20t$

$R = 20\sqrt{20}$ m

1

Q6 (v) $t = \sqrt{20}$ $\quad \dot{x} = 20$

$\dot{y} = -10(\sqrt{20})$



$v^2 = \dot{x}^2 + \dot{y}^2$

$= (20)^2 + (-10\sqrt{20})^2$

$= 400 + 2000$

$v^2 = 2400$

2

$\therefore v = \sqrt{2400}$

$v = 20\sqrt{6}$ m/s

$\tan\alpha = \dfrac{\dot{y}}{\dot{x}}$

$= \dfrac{-10\sqrt{20}}{20}$

$\tan\alpha = -\dfrac{\sqrt{20}}{2}$

$= \dfrac{-2\sqrt{5}}{2}$

$\tan\alpha = -\sqrt{5}$

$\therefore \alpha = 180 - 65°54'$

$\alpha = 114°5'$

Q7 (a) A particle is moving in SHM if
2 (1) it is travelling in a straight line and
(2) if it experiences an acceleration of the $\ddot{x} = -n^2x$

(b, (i) When $t=0$, $x=0$, $v=2p$ m/s

$$\ddot{x} = -9x$$

$$\frac{d}{dx}\left(\tfrac{1}{2}v^2\right) = -9x$$

$$\therefore \tfrac{1}{2}v^2 = -9\frac{x^2}{2} + c$$

$x=0$, $v=2p$ } $\tfrac{1}{2}(2p)^2 = -9\frac{(0)^2}{2} + c$

$$\therefore c = 2p^2$$

$$\therefore \tfrac{1}{2}v^2 = -\frac{9x^2}{2} + 2p^2$$

$$\therefore v^2 = 4p^2 - 9x^2$$

3 When it comes to rest, $v=0$.

$$\therefore 0 = 4p^2 - 9x^2$$

$$\therefore x^2 = \frac{4p^2}{9}$$

$$\therefore x = \frac{2p}{\sqrt{9}} \text{ m}$$

07 (b)(ii) $x = a\cos(nt + \alpha)$

when $t=0$, $x=0$ ∴

∴ $0 = a\cos\alpha$

∴ $\alpha = \pi/2$

∴ $x = a\cos(nt + \pi/2)$

$= a\left[\cos nt \cdot \cos\frac{\pi}{2} - \sin nt \cdot \sin\frac{\pi}{2}\right]$

2

$x = -a\sin nt$

At ~~point~~, $x = \frac{2p}{\sqrt{q}} = a$

∴ $\sin nt = -1$ when $n^2 = q$ ∴ $n = \sqrt{q}$

2

∴ $\sin(\sqrt{q}\,t) = -1$

∴ $\sqrt{q}\,t = \frac{3\pi}{2}$

∴ $t = \frac{3\pi}{2\sqrt{q}}$ sec

(iii) $t = \frac{\pi}{2\sqrt{4q}} = \frac{\pi}{4\sqrt{q}}$,

$x = -\frac{2p}{\sqrt{q}} \cdot \sin(\sqrt{q}\,t)$

3

$= -\frac{2p}{\sqrt{q}}\left(\sin \sqrt{q} \cdot \frac{\pi}{4\sqrt{q}}\right)$

$= -\frac{2p}{\sqrt{q}} \cdot \sin\pi/4 = -\frac{\sqrt{2}\,p}{\sqrt{q}} \cdot \frac{1}{\sqrt{2}}$

$x = -\frac{\sqrt{2}}{\sqrt{q}}\,p$ m